



Equilibrium

Engineering mechanics





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Equilibrium

3.1. Introduction

- **Statics** deals with rigid and semi-rigid **bodies in equilibrium**.
- Hence equilibrium constitute the most important part of statics.
- A body is said to be in **equilibrium** when the **resultant of all the forces acting on it is zero**.

$$\mathbf{R} = \Sigma \mathbf{F} = \mathbf{0} \quad \mathbf{M} = \Sigma \mathbf{M} = \mathbf{0}$$



3.2. EQUILIBRIUM IN TWO-DIMENSIONS



3.2. Equilibrium in Two Dimension

- All physical bodies are three-dimensional, but can be treated as **two-dimensional when** the forces to which they are subjected **act in a single plane**.
- When this simplification is not possible the problem will be treated as three dimensional.



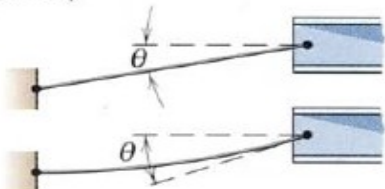
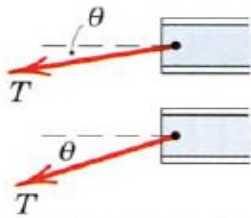
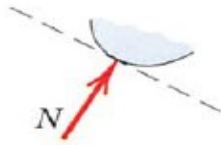
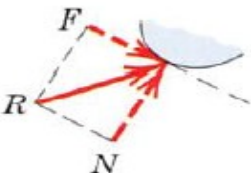
3.2.1. System Isolation and the Free Body Diagram

- Before considering equilibrium conditions
 - ✓ It is necessary to define unambiguously the particular body or mechanical system
 - ✓ and represent clearly and completely all forces which act on the body
- A mechanical system is defined as a body or group of bodies that can be conceptually isolated from all other bodies
- This isolation from surrounding bodies is accomplished by means of the free body diagram

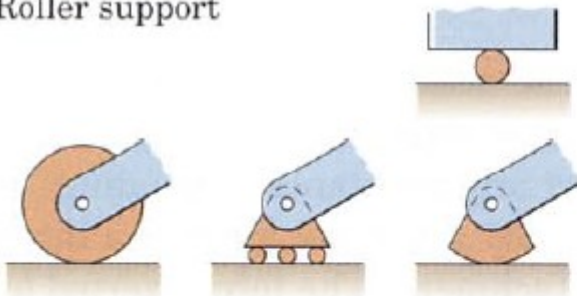
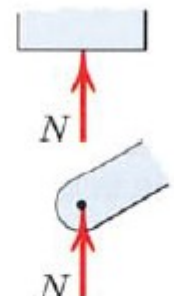
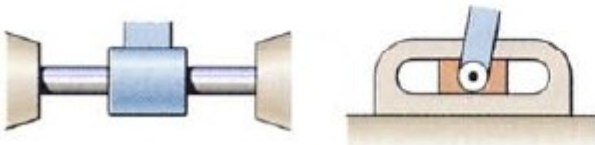
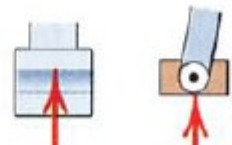
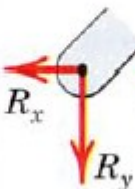
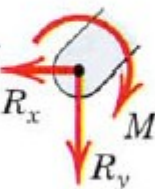
3.2.1. System Isolation and the Free Body Diagram

Modeling the action of forces

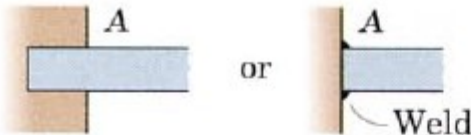
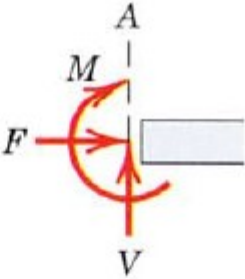
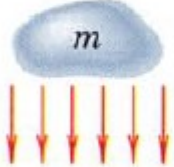
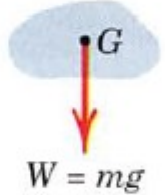
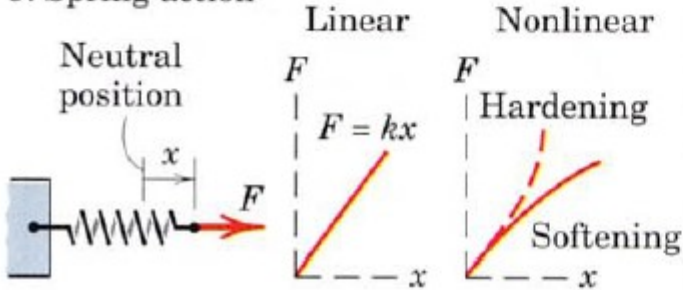
- The **most important step** in drawing the free body diagram will be to show the **external forces exerted on the**

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.

3.2.1. System Isolation and the Free Body Diagram

4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.
6. Pin connection 	Pin free to turn  Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y. A pin not free to turn may also support a couple M.

3.2.1. System Isolation and the Free Body Diagram

7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.



3.2.1. System Isolation and the Free Body Diagram

Steps for the construction of free body diagram

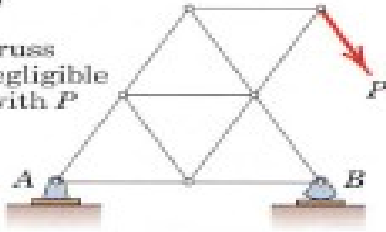
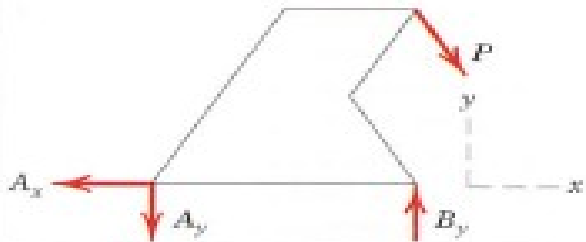
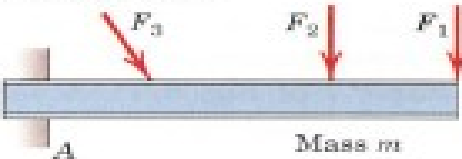
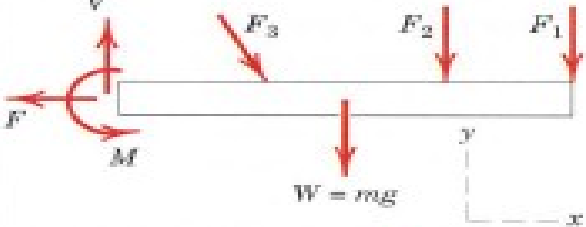
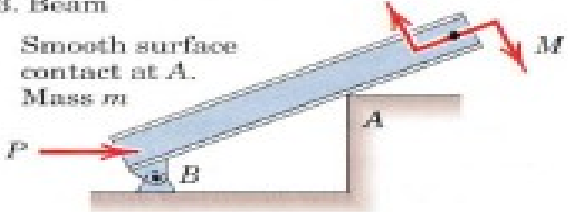
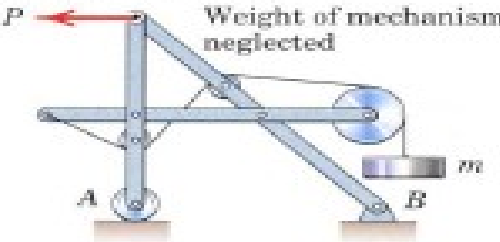
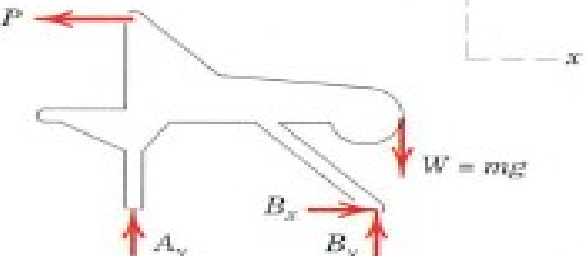
Step 1 Decide which body or combination of bodies to be considered.

Step 2 The body or combination chosen is isolated by a diagram that represents its complete external boundary.

Step 3 All forces that act on the isolated body by the removed contacting and attracting bodies and represented in their proper positions on the diagram of the isolated body.

Step 4 Show the choice of coordinate axes directly on the diagram

Examples of Free Body Diagram

SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	Free-Body Diagram of Isolated Body
1. Plane truss Weight of truss assumed negligible compared with P 	
2. Cantilever beam  Mass m	
3. Beam Smooth surface contact at A. Mass m 	
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected 	

3.2.2. Equilibrium Conditions

- It was stated that a body is in equilibrium if the resultant force vector and the resultant couple vector are both zero.

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_O = 0$$

- **Categories of Equilibrium**

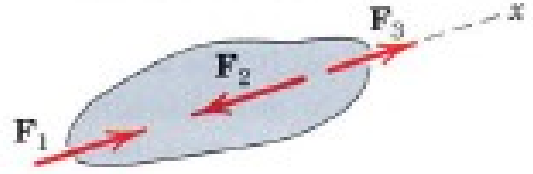
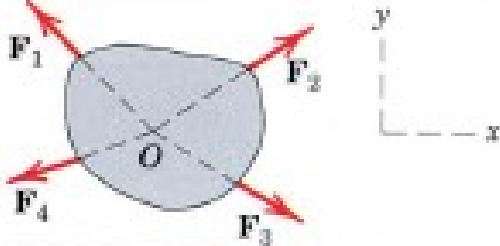
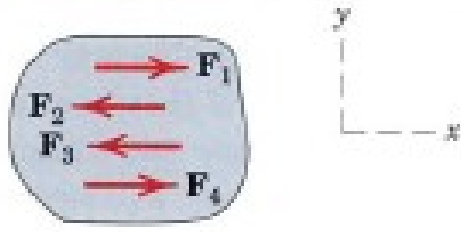
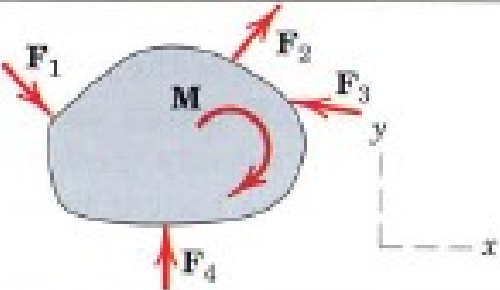
Category 1 equilibrium of **collinear** forces

Category 2 equilibrium of **concurrent** forces

Category 3 equilibrium of **parallel** forces in a plane

Category 4 equilibrium of a **general** system of forces in a plane

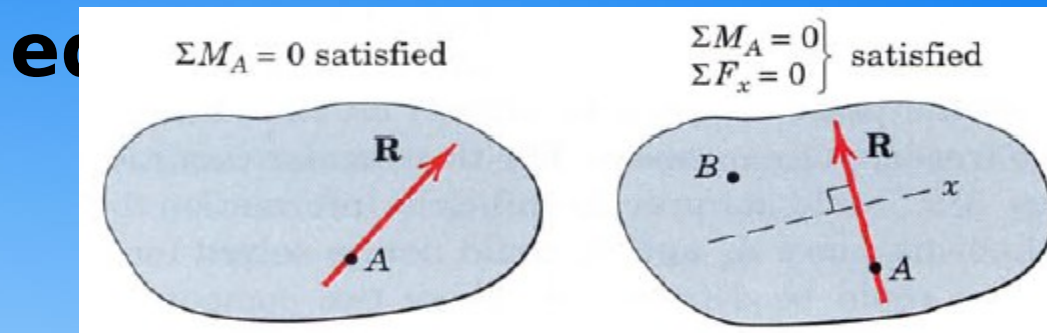
3.2.2. Equilibrium Conditions

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

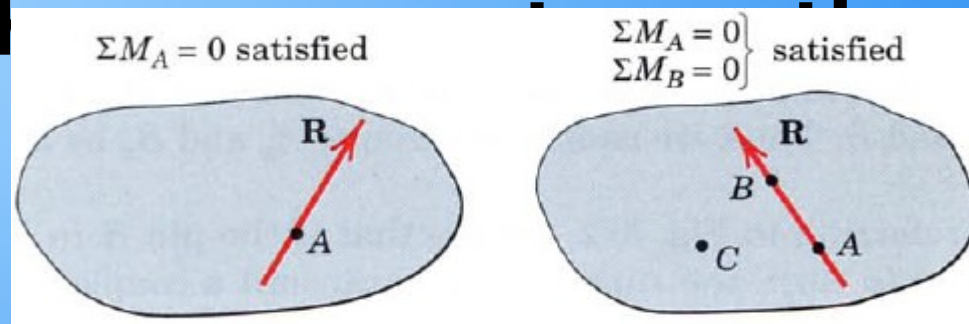
3.2.2. Equilibrium Conditions

Alternative equilibrium equations

✓ One force and two moment



✓ Two forces





3.2.2. Equilibrium Conditions

Constraints and Statical Determinacy

- The equilibrium equations developed earlier are necessary and sufficient conditions to establish the equilibrium of a body.
- But this does not necessarily provide all the information to calculate all the unknowns
- The adequacy of the equations to determine the unknowns depends



3.2.2. Equilibrium Conditions

- A rigid body or rigid combination of elements treated as a single body, which possesses **more external supports or constraints than are necessary** to maintain an equilibrium position called *statically indeterminate*.
- *Equilibrium equations are not enough to determine the unknown forces.*



3.2.2. Equilibrium Conditions

- Supports which can be removed without destroying the equilibrium condition of the body are said to be *redundant*
- The number of redundant corresponds to the degree of statical indeterminacy

Degree of statical indeterminacy =
Unknown external forces – number
of independent equilibrium
equations

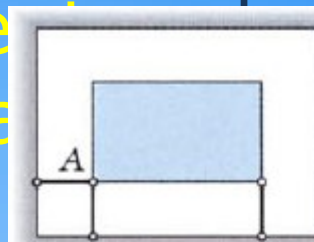


3.2.2. Equilibrium Conditions

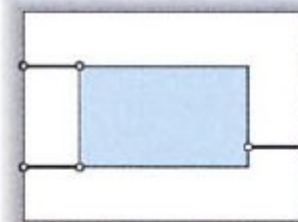
- Bodies which are supported by the minimum number of constraints necessary to ensure an equilibrium configuration are called *statically determinate*.
- *Equilibrium equations are sufficient to determine the unknown forces.*

3.2.2. Equilibrium Conditions

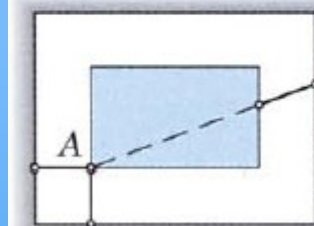
- **Adequacy of constraint**
- The **adequacy of the constraints** to prevent possible movement of the body depends on the **number**, **arrangement**, and **characteristics** of the constraints.



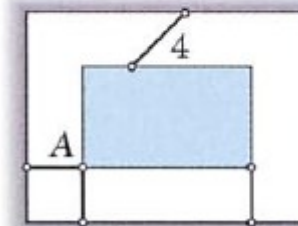
Complete fixity
Adequate constraints



Incomplete fixity
Partial constraints

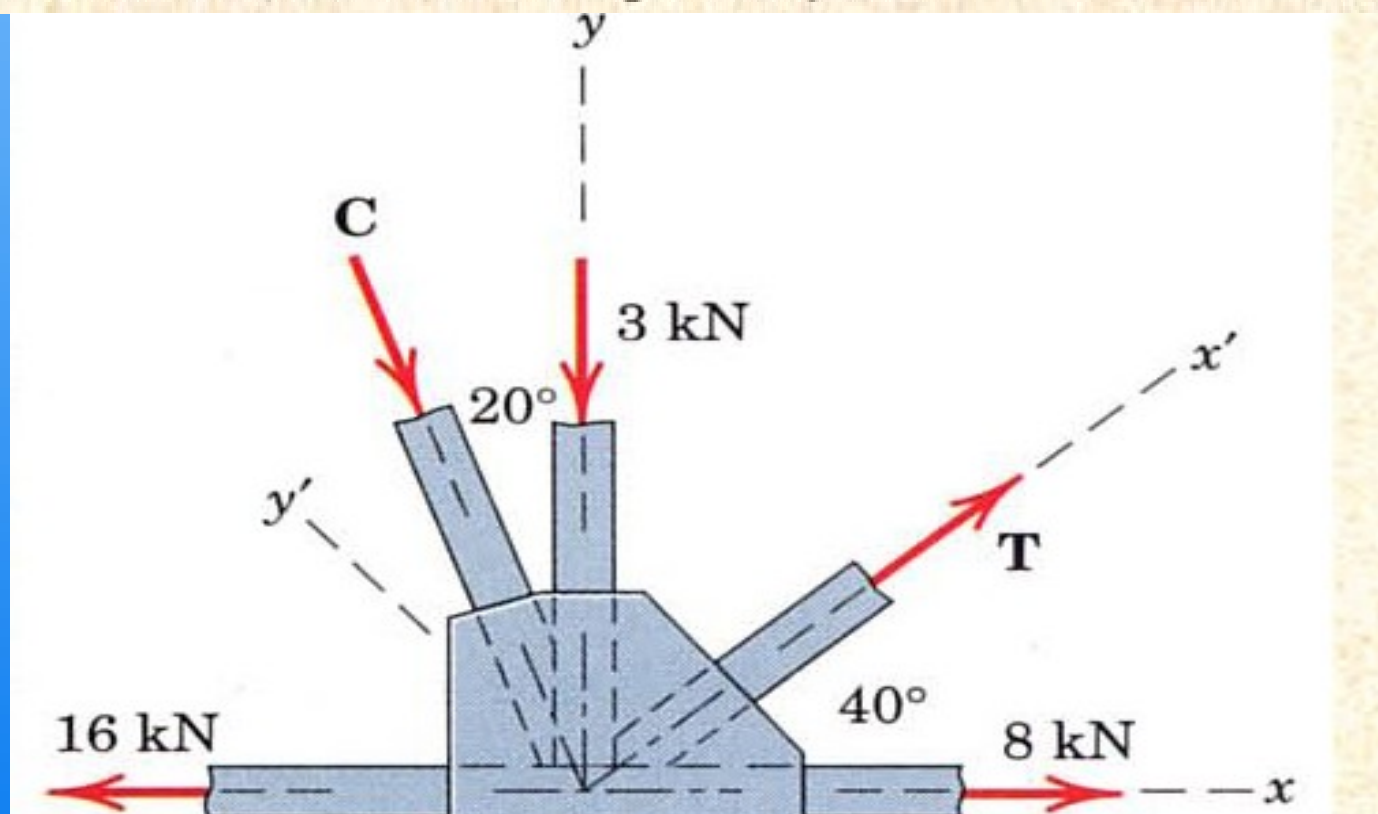


Incomplete fixity
Partial constraints



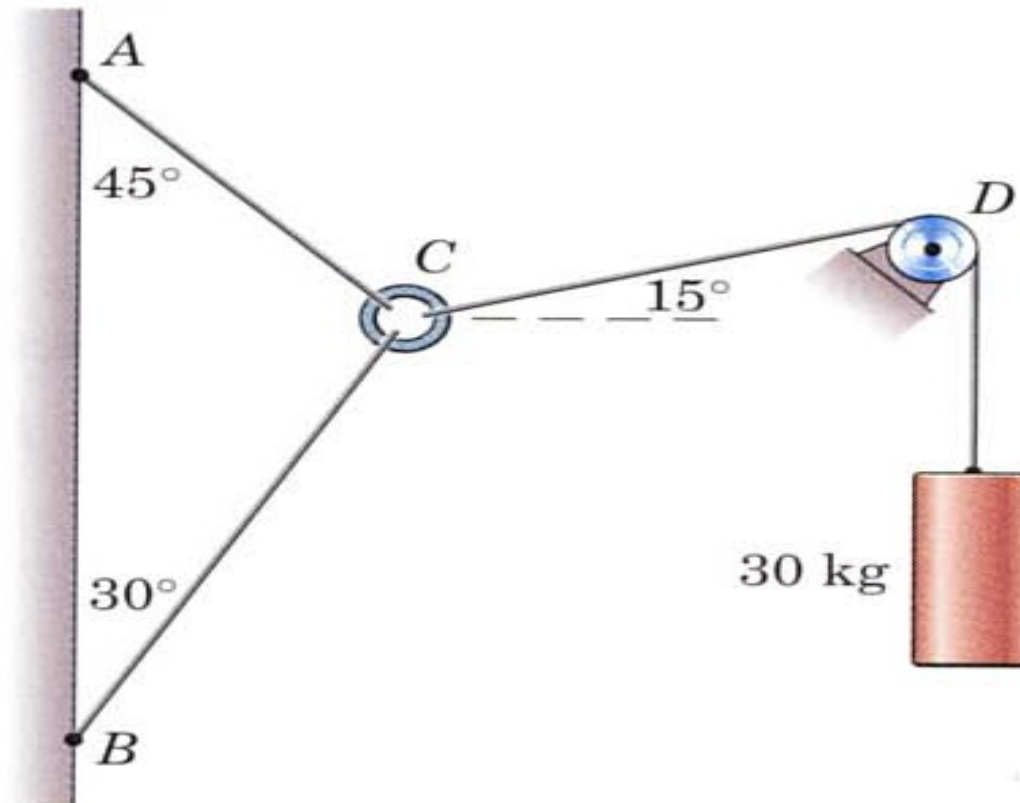
Excessive fixity
Redundant constraint

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



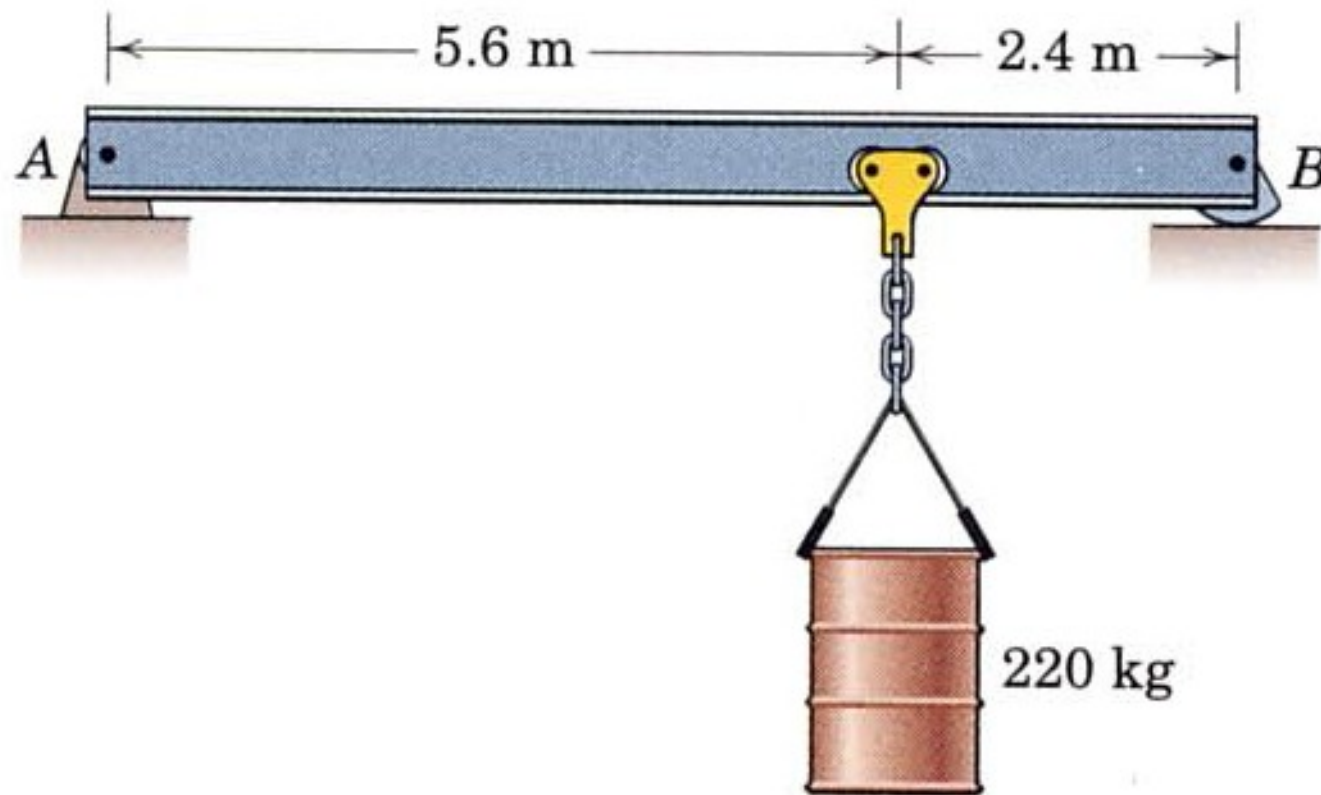
example

Three cables are joined at the junction ring C . Determine the tensions in cables AC and BC caused by the weight of the 30-kg cylinder.



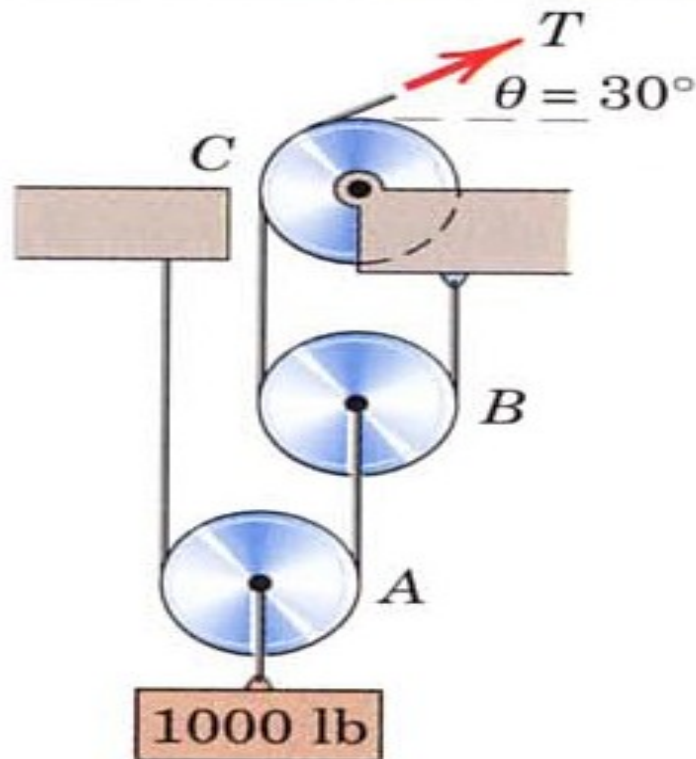
example

The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



example

Calculate the tension T in the cable which supports the 1000-lb load with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.





3.3. EQUILIBRIUM IN THREE-DIMENSIONS

3.3. Equilibrium in Three-Dimensions

3.3.1. Equilibrium conditions

- For a body to be in equilibrium it is required that the resultant force and resultant couple on a body be zero.

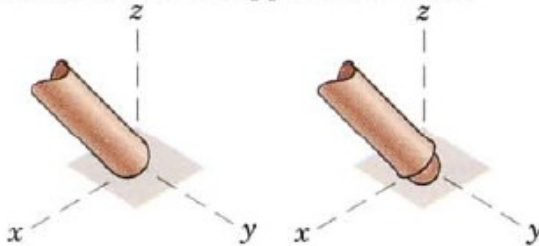
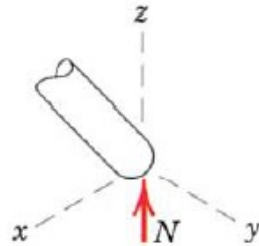
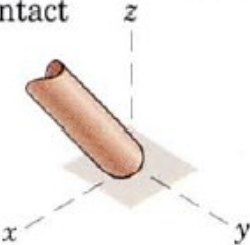
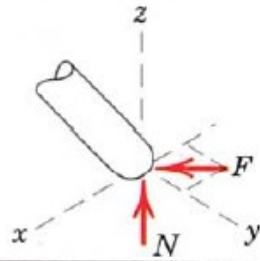
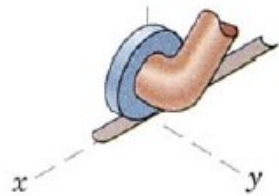
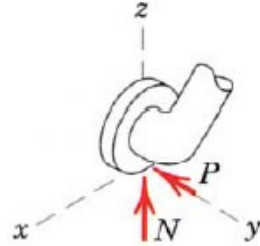
$$\begin{array}{l} \Sigma \mathbf{F} = \mathbf{0} \quad \text{or} \quad \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{cases} \\ \Sigma \mathbf{M} = \mathbf{0} \quad \text{or} \quad \begin{cases} \Sigma M_x = 0 \\ \Sigma M_y = 0 \\ \Sigma M_z = 0 \end{cases} \end{array}$$

- These six equations are both **necessary** and **sufficient** conditions for complete equilibrium.

3.3. Equilibrium in Three-Dimensions

Modeling the action of forces

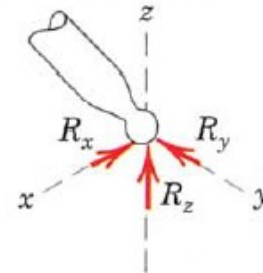
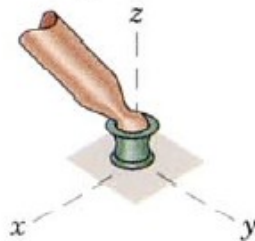
MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Member in contact with smooth surface, or ball-supported member 	 Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface 	 The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint 	 A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.

3.3. Equilibrium in Three-Dimensions

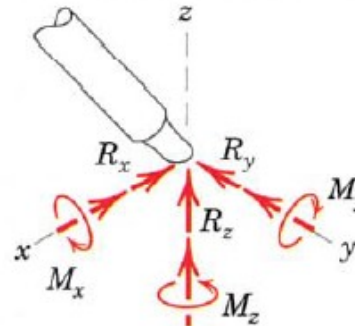
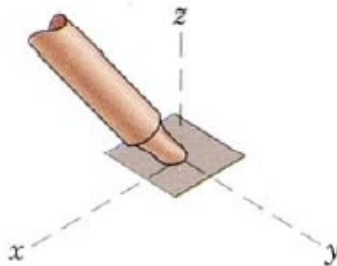
Modeling the action of forces

4. Ball-and-socket joint



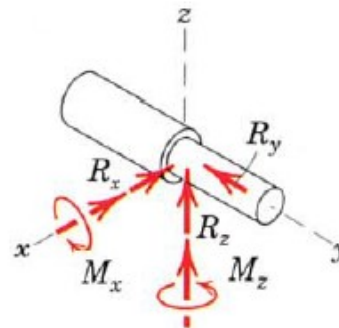
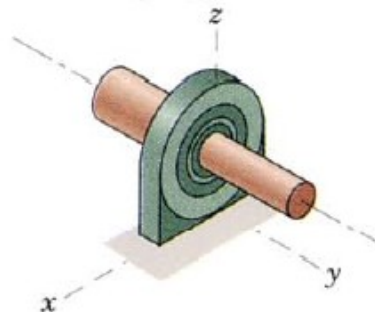
A ball-and-socket joint free to pivot about the center of the ball can support a force $\mathbf{R}$ with all three components.

5. Fixed connection (embedded or welded)



In addition to three components of force, a fixed connection can support a couple $\mathbf{M}$ represented by its three components.

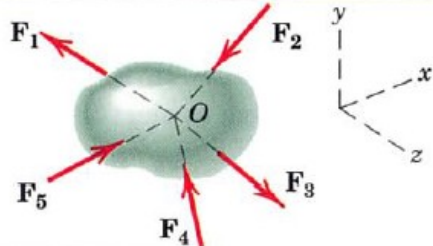
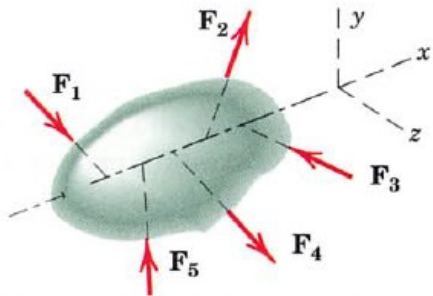
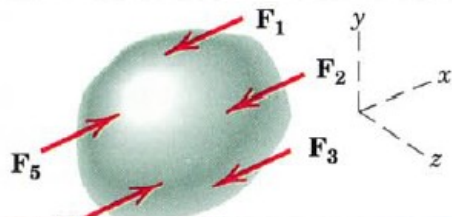
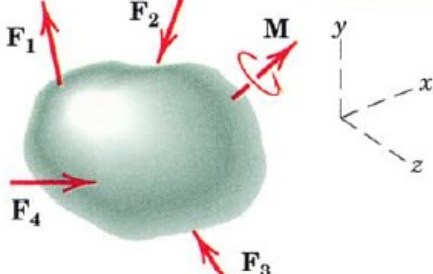
6. Thrust-bearing support



Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z . Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

3.3. Equilibrium in Three-Dimensions

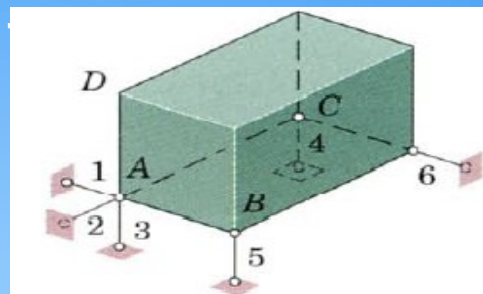
Categories of Equilibrium

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

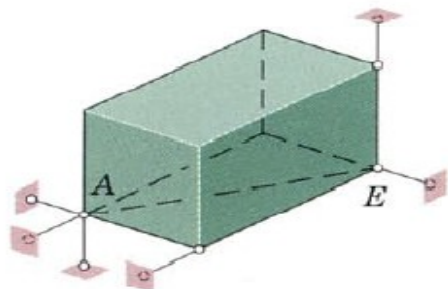
3.3. Equilibrium in Three-Dimensions

Constraints and Static Determinacy

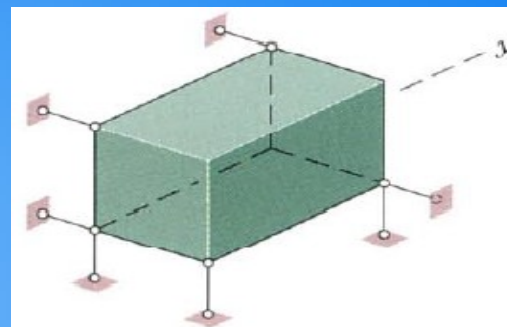
- The adequacy of the equations to determine the unknowns depends on



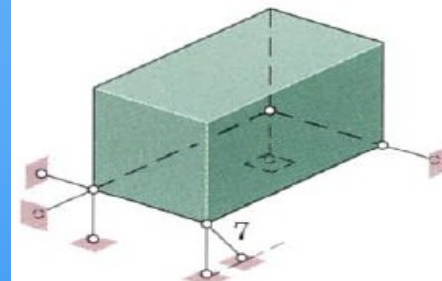
(a) Complete fixity
Adequate constraints



(b) Incomplete fixity
Partial constraints



(c) Incomplete fixity
Partial constraints



(d) Excessive fixity
Redundant constraints

forces in terms of the coordinate unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$. For the first equation, $\Sigma \mathbf{F} = \mathbf{0}$, the vector sum will be zero only if the coefficients of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ in the expression are, respectively, zero. These three sums, when each is set equal to zero, yield precisely the three scalar equations of equilibrium, $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$.

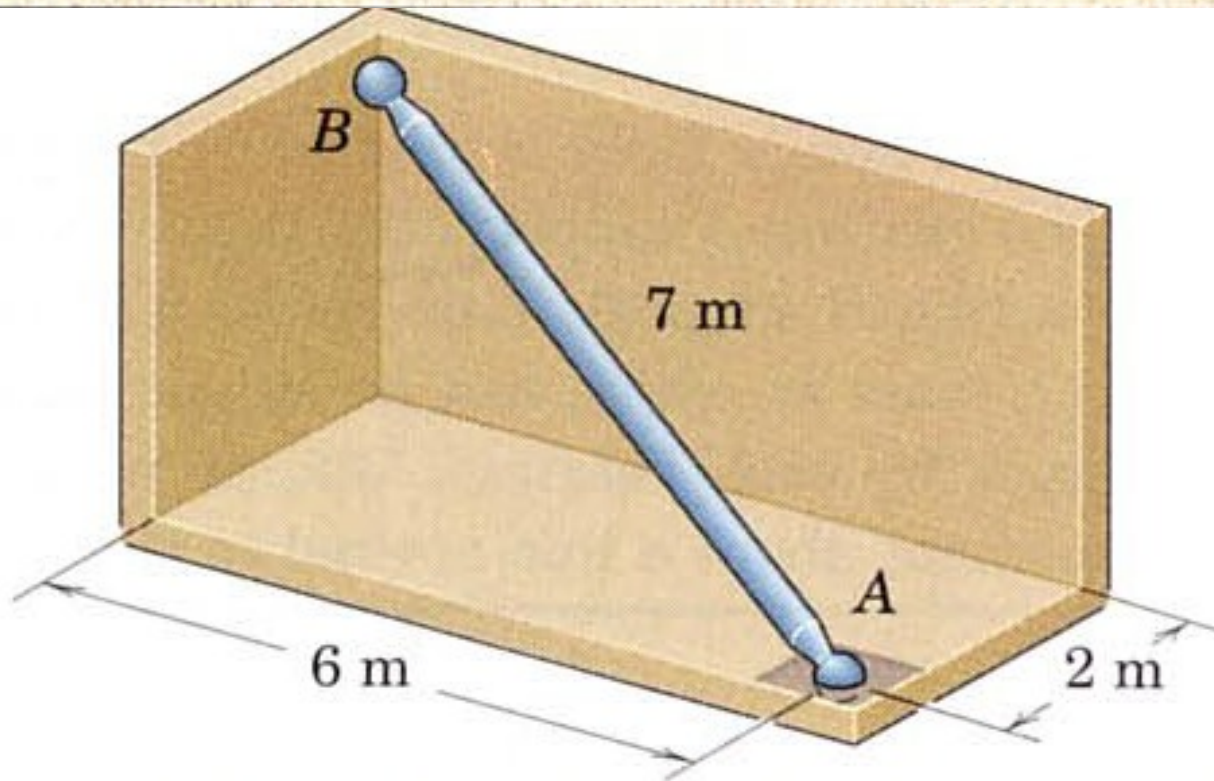
For the second equation, $\Sigma \mathbf{M} = \mathbf{0}$, where the moment sum may be taken about any convenient point O , we express the moment of each force as the cross product $\mathbf{r} \times \mathbf{F}$, where $\mathbf{r}$ is the position vector



from O to any point on the line of action of the force $\mathbf{F}$. Thus $\Sigma \mathbf{M} = \Sigma(\mathbf{r} \times \mathbf{F}) = \mathbf{0}$. When the coefficients of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ in the resulting moment equation are set equal to zero, respectively, we obtain the three scalar moment equations $\Sigma M_x = 0$, $\Sigma M_y = 0$, and $\Sigma M_z = 0$.

example

The uniform 7-m steel shaft has a mass of 200 kg and is supported by a ball-and-socket joint at A in the horizontal floor. The ball end B rests against the smooth vertical walls as shown. Compute the forces exerted by the walls and the floor on the ends of the shaft.



example

Three workers are carrying a 4-ft by 8-ft panel in the horizontal position shown. If the homogeneous panel weighs 100 lb, estimate the lifting force exerted by each worker.

